Introduction to symbolic logic

Argument and syllogism

• An argument consists of a set of premises (hypotheses), a conclusion following from those premises, and the steps taken to arrive at that conclusion.

Example:

Premise 1: On Sundays I mow the lawn.
Premise 2: Today is Sunday.
Conclusion: (Therefore) I mow the lawn.

• A valid argument is one where the structure of the argument guarantees that, if the premises are true, the conclusion is forced to follow.

Example:

Premise 1: All elephants are pink.
Premise 2: There is an elephant in my closet.
Conclusion: (Therefore) The elephant in my closet is pink.

• Valid arguments don’t have to be true arguments - validity is about structure, not truth. This argument is valid because if the premises were true (elephants really were pink and there really was one in my closet), the conclusion would have to be true. Since the premises are in fact false, the conclusion could be false as well. Or true. You can’t tell what’s going to follow from false premises.

• A syllogism is

A syllogism, also known as a rule of inference, is a formal logical scheme used to draw a conclusion from a set of premises.¹

Example:

Premise 1: All A are B.
Premise 2: x is an A.
Conclusion: (Therefore) x is a B.

• An invalid argument is one that is not guaranteed to produce a true conclusion from true premises. It’s not guaranteed to produce a false one either; the main point about invalid arguments is that any conclusion you state is accidental - it doesn’t follow in a logical way from the premises.

Example:

• It’s a bit easier to state rules / give arguments when they are in the form of implications:

\[
\begin{align*}
\text{Premise 1:} & \quad \text{All A are B.} \\
\text{Premise 2:} & \quad \text{x is an A.} \\
\hline
\text{Conclusion:} & \quad (\text{Therefore}) \text{ x is a B.}
\end{align*}
\]

becomes (with \( p = \text{“it’s an A”} \) and \( q = \text{“it’s a B”} \))

\[
\begin{align*}
\text{Premise 1:} & \quad p \rightarrow q \\
\text{Premise 2:} & \quad p \\
\hline
\text{Conclusion:} & \quad q
\end{align*}
\]

and can be expressed as a single compound statement:

\[
((p \rightarrow q) \land p) \rightarrow q
\]
Valid argument forms (named syllogisms):

<table>
<thead>
<tr>
<th>Modus Ponens</th>
<th>Modus Tollens</th>
<th>Disjunctive Syllogism</th>
<th>Transitivity*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow q$</td>
<td>$p \rightarrow q$</td>
<td>$p \lor q$ $p \lor q$</td>
<td>$p \rightarrow q$</td>
</tr>
<tr>
<td>$p$</td>
<td>$\neg q$</td>
<td>$\neg p$</td>
<td>$q \rightarrow r$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\neg p$</td>
<td>$q$</td>
<td>$p \rightarrow r$</td>
</tr>
</tbody>
</table>

*also called Hypothetical Syllogism.

Invalid argument forms (named fallacies):

<table>
<thead>
<tr>
<th>Fallacy of the Converse*</th>
<th>Fallacy of the Inverse**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow q$</td>
<td>$p \rightarrow q$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\neg p$</td>
</tr>
<tr>
<td>$p$</td>
<td>$\neg q$</td>
</tr>
</tbody>
</table>

* Also called “affirming the consequent”.
** Also called “denying the antecedent”.

- To determine whether a particular argument is valid or invalid, translate the premises and conclusion using logical operators, and identify the form.

Example: Decide if the argument is valid or invalid, and cite the syllogism / fallacy form.

If 1 + 1 = 2, then dogs bark.
If dogs bark, then cats meow.
If 1 + 1 = 2, then cats meow.
Mini-quiz: Decide if the argument is valid or invalid, and cite the syllogism / fallacy form.

If the month is May, then you’re taking a class.

(Q1) The month is May.
You’re taking a class.

(a) Valid by modus ponens.
(b) Valid by modus tollens.
(c) Valid by disjunctive syllogism.
(d) Valid by transitivity.
(e) Invalid - fallacy of the inverse.
(f) Invalid- fallacy of the converse.

If the month is May, then you’re taking a class.

(Q2) The month is June.
You’re not taking a class.

(a) Valid by modus ponens.
(b) Valid by modus tollens.
(c) Valid by disjunctive syllogism.
(d) Valid by transitivity.
(e) Invalid - fallacy of the inverse.
(f) Invalid- fallacy of the converse.

If my cat has feathers, then that donkey can swim.

(Q3) That donkey cannot swim.
My cat does not have feathers.

(a) Valid by modus ponens.
(b) Valid by modus tollens.
(c) Valid by disjunctive syllogism.
(d) Valid by transitivity.
(e) Invalid - fallacy of the inverse.
(f) Invalid- fallacy of the converse.
Either cats have tails, or dogs have fleas.

Cats do not have tails.

Dogs have fleas.

(a) Valid by modus ponens.
(b) Valid by modus tollens.
(c) Valid by disjunctive syllogism.
(d) Valid by transitivity.
(e) Invalid - fallacy of the inverse.
(f) Invalid - fallacy of the converse.

If my cat has feathers, then that donkey can swim.

That donkey can swim.

My cat has feathers.

(a) Valid by modus ponens.
(b) Valid by modus tollens.
(c) Valid by disjunctive syllogism.
(d) Valid by transitivity.
(e) Invalid - fallacy of the inverse.
(f) Invalid - fallacy of the converse.