Vector fields, line integrals, and Green's Theorem

grad, div, curl – suggested problems

P1. Find the gradient vector field for the scalar function \( f(x, y) = 5x^2 + 3xy + 10y^2 \).

P2. Find the divergence div \( \mathbf{F} \) for

\[
\mathbf{F}(x, y) = xe^y \mathbf{i} + ye^y \mathbf{j}
\]

P3. Find the divergence div \( \mathbf{F} \) for

\[
\mathbf{F}(x, y, z) = x^2 z \mathbf{i} - 2xz \mathbf{j} + yz \mathbf{k}
\]

and evaluate at the point \((2, -1, 3)\).

P4: Find the curl, curl \( \mathbf{F} \) for

\[
\mathbf{F}(x, y, z) = x^2 z \mathbf{i} - 2xz \mathbf{j} + yz \mathbf{k}
\]

and evaluate at the point \((2, -1, 3)\).

P5: For the functions

\[
\mathbf{F}(x, y, z) = \mathbf{i} + 2x \mathbf{j} + 3y \mathbf{k}
\]
\[
\mathbf{G}(x, y, z) = x \mathbf{i} - y \mathbf{j} + z \mathbf{k}
\]

The point to this one is that we can stack operators (as long as it's defined on the vector or scalar functions as appropriate). I could invent a dozen variations; expect to see some more on the assignment. Just do whatever the notation calls for...

Find

(a) \( \text{curl}(\mathbf{F} \times \mathbf{G}) = \nabla \times (\mathbf{F} \times \mathbf{G}) \)
(b) \( \text{div}(\mathbf{F} \times \mathbf{G}) = \nabla \cdot (\mathbf{F} \times \mathbf{G}) \)
(c) \( \text{grad}(\text{div}(\mathbf{F} \times \mathbf{G})) = \nabla \left( \nabla \cdot (\mathbf{F} \times \mathbf{G}) \right) \)