Vector fields, line integrals, and Green's Theorem

Vector fields – suggested problems

P1: Give five vectors which are in the vector field

\[ \mathbf{F}(x, y, z) = x^2 \mathbf{i} - (2xy + z) \mathbf{j} + (\sin z) \mathbf{k} \]

This is totally random – just pick a bunch of \((x, y, z)\) triplets and have at it. Notice that some triplets will produce the same vectors, but they aren’t really duplicates if you go to graph them, since they’re coming out of different points. However, shoot for five distinct vectors.

P2: For the vector field \( \mathbf{F}(x, y) = y\mathbf{i} + 2x\mathbf{j} \)

(a) Write the expression for \( \| \mathbf{F}(x, y) \| \).
(b) For \( c = 1, \ c = 2, \) and \( c = 3 \), write expressions for the curves \( \| \mathbf{F}(x, y) \|^2 = c^2 \).
(c) Describe and sketch the above curves. These are simple enough that you can do them by hand.
(d) Take the curve \( \| \mathbf{F}(x, y) \|^2 = c^2 \) for each of the above values of \( c \) and construct a table of various \((x, y)\) pairs that lie on that curve, and find the value of \( \mathbf{F} \) at each of these points. Add the vectors to the curves. Do enough of them so you have a feel for what the field looks like. This is tedious, but within the scope of still doable by hand – you’ll probably see a pattern emerge that gives you an overall sense of the field.
(e) Use some sort of plotting software (MVT, Maple) to produce a plot of the vector field. Recall that in the software plots, the vectors are scaled down to reduce overcrowding.
(f) Imagine that the field represents some sort of flow (water, air) and drop objects in at the points \((2, 0)\) and \((1, -3)\). On top of your software generated plot, chart the course the objects would take.

P3: Use MVT or Maple to plot the vector fields in the plane:

(a) \( \mathbf{F}(x, y) = \frac{1}{8} (2xy \mathbf{i} + y^2 \mathbf{j}) \)
(b) \( \mathbf{F}(x, y) = 15y^2 \mathbf{i} - 5xy^2 \mathbf{j} \)
P4: Use MVT or Maple to plot the vector fields in space.

(a) \( \mathbf{F}(x, y, z) = \frac{x \mathbf{i} + y \mathbf{j} + z \mathbf{k}}{\sqrt{x^2 + y^2 + z^2}} \)

(b) \( \mathbf{F}(x, y, z) = \frac{yz}{y - z} \mathbf{i} + \frac{xz}{x - z} \mathbf{j} + \frac{xy}{x - y} \mathbf{k} \)