Unit vectors and direction

- Any vector of length (magnitude) 1 is a **unit vector**. Given a vector \( \mathbf{v} \), you can construct a unit vector \( \mathbf{u} \) in the same direction as \( \mathbf{v} \) by normalizing \( \mathbf{v} \):

\[
\mathbf{u} = \frac{1}{||\mathbf{v}||} \mathbf{v}
\]

**Example:** Normalize \( \mathbf{v} = <1, -1, 5> \).

- A unit vector can be used to represent the **direction** of a given vector - normalizing \( \mathbf{v} \) gives us a way to borrow its direction, without borrowing its magnitude. To find a vector \( \mathbf{w} \) with a given magnitude, and in the direction of a given vector \( \mathbf{v} \),

  * Find the unit vector in the direction of \( \mathbf{v} \).
  * Multiply the unit vector by the given magnitude, to scale it to the correct length.

**Example:** Find a vector \( \mathbf{w} \) in the direction of \( \mathbf{v} = <2, 3> \), but with \( ||\mathbf{w}|| = 5 \).
• In the plane, the vectors $\mathbf{i} = <1, 0>$ and $\mathbf{j} = <0, 1>$ are the \textbf{standard unit vectors}. In space, the standard unit vectors are $\mathbf{i} = <1, 0, 0>$, $\mathbf{j} = <0, 1, 0>$, and $\mathbf{k} = <0, 0, 1>$.

• Any vector can be written as a linear combination of the standard unit vectors; for example
  
  $<2, -3, 4> = 2 <1, 0, 0> - 3 <0, 1, 0> + 4 <0, 0, 1> = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

• The vectors $\mathbf{i}$ and $\mathbf{j}$ are referred to as the \textbf{standard basis} for $\mathbb{R}^2$ (they span and are independent). $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the standard basis for $\mathbb{R}^3$, and so on. The idea of basis is covered in detail in Linear Algebra.

\textbf{Example:} Express $<-2, 0, -4>$ as a linear combination of the standard basis vectors.

\textbf{Example:} Calculate $(3\mathbf{i} - 2\mathbf{k}) - 3(\mathbf{i} + \mathbf{j} + 5\mathbf{k})$