Entering and plotting vector fields in Maple

To get a vector field entered into Maple, you need to load the VectorCalculus package, *and* apparently the Student[VectorCalculus] package if you want the thing to plot, so just to cover all bases, start off with

\[
\text{with(VectorCalculus)};
\]
\[
\text{with(Student[VectorCalculus])}:
\]
\[
\text{with(plots)}:
\]

To enter a function, say \( F(x, y) = x^2 y \mathbf{i} + (x - y) \mathbf{j} \), you need to first set the coordinate system and enter as shown.

\[
\text{SetCoordinates(cartesian[x,y]);}
\]
\[
\begin{align*}
\text{cartesian}_{x,y} \\
F := \text{VectorField}(x^2 \cdot y, x - y); \\
(x^2 \cdot y)\mathbf{e}_x + (x - y)\mathbf{e}_y
\end{align*}
\]

The command evalVF can be used to evaluate a vector field at a point:

\[
\text{evalVF}(F, 2, 5); \\
\begin{bmatrix}
20 \\
-3
\end{bmatrix}
\]

(evaluated the function at \((x, y) = (2, 5)\))

And the plotting command is, oddly, still VectorField - you just need to specify output=plot. Play around with the grid size to get more or less vectors showing in your field. One thing to notice about vector field plots is that usually the magnitudes of the vectors are relative to each other, but not sized absolutely in the same scale as the grid (if the vector at \((2, 5)\) were really displayed as \((20, -3)\), it'd be well off the page. Instead, it's proportioned correctly in a 20 (x) to -3 (y) ratio, and scaled relative to the other vectors in the field.

\[
\text{VectorField}(F, \text{output} = \text{plot}, \text{view} = [-5..5, -5..5], \text{fieldoptions} = [\text{grid} = [10, 10], \text{thickness} = 2]);
\]
Student[VectorCalculus] also lets you plot and even animate **flow lines** - pick a starting point, and you can get a curve that travels in the direction indicated by the vectors. Interpreting a vector field in this way comes up in Differential Equations - the vectors are interpreted as a **slope field**, and any initial condition defines a scalar function by "start here and travel in the direction indicated by the vectors."

```plaintext
FlowLine(F, [.1, .1], output = plot, view = [ -5 ..5, -5 ..5 ], fieldoptions = [ grid = [10, 10], thickness = 2 ]); 
```
Using the display command to get multiple flow lines...

\[ Fp1 := \text{FlowLine}(F, (1, 1), \text{output} = \text{plot}, \text{view} = [-5..5, -5..5], \text{fieldoptions} = [\text{grid} = [10, 10], \text{thickness} = 2]) ; \]

\[ Fp2 := \text{FlowLine}(F, (-3, 1), \text{output} = \text{plot}, \text{view} = [-5..5, -5..5], \text{fieldoptions} = [\text{grid} = [10, 10], \text{thickness} = 2]) ; \]

\[ Fp3 := \text{FlowLine}(F, (-1, -1), \text{output} = \text{plot}, \text{view} = [-5..5, -5..5], \text{fieldoptions} = [\text{grid} = [10, 10], \text{thickness} = 2]) ; \]

\[ \text{display}([Fp1, Fp2, Fp3]) ; \]
Example of a 3d vector field

The next one is a plot of \( \mathbf{F}(x, y, z) = x^2 y \mathbf{i} + (x - y + z) \mathbf{j} + (2x - z) \mathbf{k} \).

SetCoordinates(cartesian[x, y, z]);

\[ \mathbf{F} := \text{VectorField}( \langle x^2 y, x - y + z, 2x - z \rangle ); \]

\[ \left( x^2 y \right) \hat{\mathbf{x}} + (x - y + z) \hat{\mathbf{y}} + (2x - z) \hat{\mathbf{z}} \]

VectorField(F, output=plot, view=[-5 .. 5, -5 .. 5, -5 .. 5], fieldoptions=[grid=[6, 6, 6], thickness=2], axes=normal, labels=[x, y, z]);
FlowLine(F, ⟨1, 1⟩, output = plot, view = [−5..5, −5..5, −5..5], fieldoptions = [grid = [6, 6, 6], thickness = 2], axes = normal, labels = [x, y, z]);