Let $p(x)$ and $q(x)$ be polynomials neither of which contain the factor $(x - a)$.

1. $r(x) = \frac{p(x)(x - a)}{q(x)}$ has an x intercept at $x = a$.
   
   $r(x)$ has the factor $(x - a)$ in the numerator but not in the denominator.

2. $r(x) = \frac{p(x)}{q(x)(x - a)}$ creates an infinite discontinuity at $x = a$.
   
   $r(x)$ has the factor $(x - a)$ in the denominator but not in the numerator.

3. $r(x) = \frac{p(x)(x - a)}{q(x)(x - a)}$ creates a removable gap at $x = a$.
   
   $r(x)$ has the factor $(x - a)$ in both the numerator and the denominator.

Example

\[ r(x) = \frac{(x + 3)(x + 1)}{(x + 2)(x + 1)} \]

Summary

- Every root upstairs is an x intercept
- Every root downstairs is a vertical asymptote
- Unless it’s the same root top and bottom which punches a hole