Determinants

Introducing determinants - definitions, and the determinant of a 2 by 2 matrix - suggested problems - solutions

P1: Compute \( \det (A) \) for \( A = \begin{bmatrix} -3 & 5 \\ 2 & -7 \end{bmatrix} \).

\[
\det(A) = -3(-7) - 5(2) = 21 - 10 = 11
\]

P2: Compute \( \begin{vmatrix} 10 & 1 \\ -5 & -4 \end{vmatrix} \).

\[
\begin{vmatrix} 10 & 1 \\ -5 & -4 \end{vmatrix} = 10(-4) - (-5)(1) = -40 + 5 = -35
\]

P3: What is the area of a parallelogram with edges defined by the vectors \( <4, -3> \) and \( <-5, -7> \) (this parallelogram would have three vertices at \((0, 0)\), \((4, -3)\), \((-5, -7)\), and the fourth vertex wherever needed to make the sides parallel - you can sketch and work that out, if you like).

Arrange vertex vectors in columns and compute \( \begin{vmatrix} 4 & -5 \\ -3 & -7 \end{vmatrix} \):

\[
\begin{vmatrix} 4 & -5 \\ -3 & -7 \end{vmatrix} = 4(-7) - (-5)(-3) = -28 - 15 = -43
\]

The area of the parallelogram is 43 square units (if answering an area question, I would give the absolute value, not the signed value).

P4: What values of \( x \) would make \( |A| = 0 \), when

\[
A = \begin{bmatrix} x - 1 & 2 \\ 2 & x + 2 \end{bmatrix}
\]

\[
|A| = (x - 1)(x + 2) - (2)(2) = x^2 + x - 2 - 4 = x^2 + x - 6
\]

\( |A| = 0 \) when \( x^2 + x - 6 = 0 \); factoring gives

\[
(x + 3)(x - 2) = 0
\]

so \( |A| = 0 \) when \( x = -3 \) or \( x = 2 \).
P5: What values of $x$ would make $|A| = 0$ when

$$A = \begin{bmatrix} x & 1 \\ -6 & x + 4 \end{bmatrix}$$

$$|A| = x(x + 4) - (1)(-6) = x^2 + 4x + 6$$

You’ll need the quadratic formula to solve $x^2 + 4x + 6 = 0$:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(6)}}{2(1)} = \frac{-4 \pm \sqrt{-8}}{2} = \frac{-4 \pm 2i\sqrt{2}}{2} = -2 \pm i\sqrt{2}$$

$|A| = 0$ when $x = -2 + i\sqrt{2}$ or $x = -2 - i\sqrt{2}$; i.e. the complex valued matrices

$$\begin{bmatrix} -2 + i\sqrt{2} & 1 \\ -6 & -2 + i\sqrt{2} + 4 \end{bmatrix} \text{ and } \begin{bmatrix} -2 - i\sqrt{2} & 1 \\ -6 & -2 - i\sqrt{2} + 4 \end{bmatrix}$$

both have zero determinants. Determinants of complex valued matrices are computed exactly as with real valued. We won’t see too many of these in this section, but keep them in mind - they’ll show up when we start working with eigenvalues.