Suggested problems

Matrix algebra properties

P1: If $A$ is a $2 \times 2$ matrix with $|A| = 3$, what are the following?
   
   (a) $|-5A|$
   (b) $|4A^t|$
   (c) $|(A^3)^{-1}|$

P2: Suppose $A$ and $B$ are $3 \times 3$ matrices with $|A| = 5$, $|B| = -3$. What are the following determinants?
   
   (a) $|2A^2B|$
   (b) $|(AB)^t|$
   (c) $|4AB^{-1}|$
   (d) $|(4AB)^{-1}|$

P3: Let $A$ be a square matrix. Prove that if there exists a positive integer $n$ such that $A^n = 0$ (that’s the zero matrix, by the way), then $|A| = 0$.

P4: Prove that if $A$ and $B$ are square matrices of the same size, then $|AB| = |BA|$.

P5: Prove that if $A$, $B$, and $C$ are square matrices of the same size, then $|ABC| = |A||B||C|$.

P6: Construct a counterexample that shows that in general, $|A + B| = |A| + |B|$ is a false statement.