Suggested problems - solutions

Applications of linear systems: circuits

Your setup may not be exactly the same, but the end result should be. Original assigned current directions may differ - I generally go through a battery in the “right” direction (short to long) if there is one in a branch, but sometimes you just have to guess.

P1:

Start by assigning currents and directions. Set up the junction equations: point D has $I_1$ in, $I_2$ and $I_3$ out, point B has $I_1$ out, $I_2$ and $I_3$ in. Either way, $I_1 = I_2 + I_3$, or

\[ I_1 - I_2 - I_3 = 0 \]

Loop ADBA (walking on this loop is walking with the assigned currents though all parts):

\[ 2I_1 + 7I_2 = 8 \]

Loop DBCD (walking on this loop is walking against $I_3$ through the 1Ω resistor):

\[ 7I_2 - 1I_3 = 0 \]

(notice no battery on this loop)

That’s enough to solve - three unknowns requires three independent equations - one junction plus two loops. The system is

\[
\begin{align*}
I_1 - I_2 - I_3 &= 0 \\
2I_1 + 7I_2 &= 8 \\
7I_2 - 1I_3 &= 0
\end{align*}
\]

SciLab:

\[
\text{-->} \text{A} = [1 -1 -1 0; 2 7 0 8; 0 7 -1 0];
\]

\[
\text{-->} \text{rref} \text{(A)}
\]

\[
\text{ans} =
\begin{align*}
1. & & 0. & & 0. & & 2.7826087 \\
0. & & 1. & & 0. & & 0.3478261 \\
0. & & 0. & & 1. & & 2.4347826
\end{align*}
\]

\[
\begin{align*}
I_1 &= 2.7826A \\
I_2 &= .3478A \\
I_3 &= 2.4348A
\end{align*}
\]
P2:

Start by assigning currents and directions. Set up the junction equations: point $B$ has $I_1$ and $I_3$ in, $I_2$ out, point $D$ has $I_1$ and $I_3$ out, $I_2$ in. Either way, $I_1 + I_3 = I_2$, or

$$I_1 - I_2 + I_3 = 0$$

Loop ABDA (walking on this loop is walking with the assigned currents though all parts):

$$2I_1 + I_2 + 4I_2 = 5$$
$$2I_1 + 5I_2 = 5$$

Loop BDCB (walking on this loop is walking with the assigned currents though all parts):

$$I_2 + 4I_2 = 6$$
$$5I_2 = 6$$

(notice $I_3$ doesn’t go through a resistor anywhere)

That’s enough to solve - three unknowns requires three independent equations - one junction plus two loops. The system is

$$I_1 - I_2 + I_3 = 0$$
$$2I_1 + 5I_2 = 5$$
$$5I_2 = 6$$

and you could easily hand solve if you wanted. SciLab is

```matlab
--> A = [1 -1 1 0; 2 5 0 5; 0 5 0 6];
--> rref(A)
ans =
   1.  0.  0. -0.5
   0.  1.  0.  1.2
   0.  0.  1.  1.7
```

and I’ve got the wrong direction on $I_1$. Solution with correct direction shown:

$$I_1 = .5A$$
$$I_2 = 1.2A$$
$$I_3 = 1.7A$$
P3:

This is a bit more involved (there are six branches in the circuit, depending on how you number). Be sure to get all possible junction equations, and fill in the rest with loops.

Start by assigning currents and directions. This is completely arbitrary, and don’t worry if something looks odd (like everything going out and nothing going in), it’ll take care of itself at the end. Set up the junction equations:

Point B has $I_1, I_2, I_3$ out, nothing in:
\[ I_1 + I_2 + I_3 = 0 \]

Point F has $I_1, I_2, I_6$ in, nothing out:
\[ I_1 + I_2 + I_6 = 0 \]

Point C has $I_3$ in, $I_4, I_5$ out:
\[ I_3 - I_4 - I_5 = 0 \]

Point E has $I_6$ out, $I_4, I_5$ in:
\[ I_4 + I_5 - I_6 = 0 \]

That’s four junction equations ... a couple of those might be redundant though. You may have already guessed that $I_3$ and $I_6$ are the same current. I’ll set up the three obvious loops, and you can see what happens when you solve the system with seven equations...

Loop AFBA (with $I_1$, with the 9V battery, against $I_2$, against the 4V battery):
\[ 2I_1 - 3I_2 = 9 - 4 \]
\[ 2I_1 - 3I_2 = 5 \]

Loop BFECB (with the battery, with $I_2$, against the others):
\[ 3I_2 - 1I_3 - 2I_4 = 4 \]

(notice $I_6$ doesn’t go through a resistor anywhere)

Loop CEDC (no battery, with $I_4$, against $I_5$):
\[ 2I_4 - 3I_5 = 0 \]
The system is

\[
\begin{align*}
I_1 + I_2 + I_3 &= 0 \\
I_1 &+ I_2 + I_6 = 0 \\
I_3 - I_4 - I_5 &= 0 \\
I_4 + I_5 - I_6 &= 0 \\
2I_1 - 3I_2 &= 5 \\
3I_2 - I_3 - 2I_4 &= 4 \\
2I_4 - 3I_5 &= 0 \\
\end{align*}
\]

That's seven equations with only six unknowns, but we're expecting a unique solution. Hopefully, a row will drop.

SciLab:

```
-->A = [1 1 0 0 0 0; 1 1 0 0 1 0; 0 0 1 -1 -1 0; 0 0 0 1 1 -1 0; 
    2 -3 0 0 0 0 5; 0 3 -1 -2 0 0 4; 0 0 0 2 -3 0 0]
A =
1. 1. 0. 0. 0. 0. 0. 
1. 1. 0. 0. 0. 1. 0. 
0. 0. 1. -1. -1. 0. 0. 
0. 0. 0. 1. 1. -1. 0. 
2. -3. 0. 0. 0. 0. 5. 
0. 3. -1. -2. 0. 0. 4. 
0. 0. 0. 2. -3. 0. 0. 

-->rref(A)
ans =
1. 0. 0. 0. 0. 0. 2.2352941 
0. 1. 0. 0. 0. 0. -0.1764706 
0. 0. 1. 0. 0. 0. -2.0588235 
0. 0. 0. 1. 0. 0. -1.2352941 
0. 0. 0. 0. 1. 0. -0.8235294 
0. 0. 0. 0. 0. 1. -2.0588235 
0. 0. 0. 0. 0. 0. 0. 
```

Yes! (Worked on the first try) You can pick the solutions off of that, and notice that most of the currents are running in the opposite direction from the initial guess.