Suggested problems

Classifying solutions: unique, none, infinitely many

In P1 through P5, use SciLab to put the system in reduced row echelon form. Then, give the solution to the system (if infinitely many, be sure to write the parametrized solutions).

P1:

\[
\begin{align*}
      x_1 - x_2 + 2x_3 &= 7 \\
      2x_1 - 2x_2 + 2x_3 - 4x_4 &= 12 \\
     -x_1 + x_2 - x_3 + 2x_4 &= -4 \\
      3x_1 + x_2 - 8x_3 - 10x_4 &= -29 \\
\end{align*}
\]

P2:

\[
\begin{align*}
    x_1 + 2x_2 &= 1 \\
   -4x_1 &= 5 \\
    3x_1 + 2x_2 &= 0 \\
\end{align*}
\]

P3:

\[
\begin{align*}
      2x_1 - 4x_2 + 16x_3 - 14x_4 &= 10 \\
     -x_1 + 5x_2 - 17x_3 + 19x_4 &= -2 \\
      x_1 - 3x_2 + 11x_3 - 11x_4 &= 4 \\
      3x_1 - 4x_2 + 18x_3 - 13x_4 &= 17 \\
\end{align*}
\]

P4:

\[
\begin{align*}
      4x_1 - 4x_2 + x_3 - x_4 &= 10 \\
     -x_1 + 5x_2 - 12x_3 + 15x_4 &= -3 \\
\end{align*}
\]

P5:

\[
\begin{align*}
    x_1 + x_2 - 2x_3 &= -1 \\
    3x_1 + 4x_2 - 2x_3 &= 0 \\
    x_1 - x_2 + x_3 &= -4 \\
    2x_1 - x_3 &= -5 \\
\end{align*}
\]

P6: What are the conditions on \(a\), \(b\), \(c\), and \(d\) for the matrix \[
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}
\] to have reduced row echelon form \[
\begin{bmatrix}
  1 & 0 \\
  0 & 1
\end{bmatrix}
\]? Note you’ll have to put the matrix into reduced row echelon form by hand; you don’t have any numbers to work with!

P7: What are the conditions on \(a\) and \(b\) needed for the system

\[
\begin{align*}
    x_1 + 2x_2 + ax_3 &= 3b \\
    x_1 + 3x_2 + ax_3 &= 4b \\
    2x_2 + ax_3 &= 3b \\
\end{align*}
\]

to have

– a unique solution?
– no solution?
– infinitely many solutions?

What is the form of the solution in case 1 and case 3?