Limits, part 1: the intuitive approach - suggested problems

In P1 - P4, find each of the following limits. Justify your answer.

P1:

\[ \lim_{(x,y) \to (0,3)} xy \cos(x - 2y) \]

P2:

\[ \lim_{(x,y) \to (1,1)} \frac{x^3 + xy^2}{x^2 + y^2} \]

P3:

\[ \lim_{(x,y) \to (0,0)} \frac{x^3 + xy^2}{x^2 + y^2} \]

P4:  (Note all of this stuff extends to functions of three or more variables.)

\[ \lim_{(x,y,z) \to (0,0,0)} e^{x^2z} \tan(y + z + 1) \]

In P5, explain why the limit does not exist by discussing the behavior of the function near (0, 0). Think about what you know about the form C \(\frac{x}{0}\) from single variable Calculus. It’s still true.

P5:

\[ \lim_{(x,y) \to (0,0)} \frac{x + 1}{xy^2} \]

In P6 - P8, describe and sketch the set in the xy plane on which the function is continuous.

P6:

\[ f(x, y) = \sin(y \ln x) \]

P7:

\[ f(x, y) = \frac{1}{\sqrt{x^2 - y}} \]

P8:

\[ f(x, y) = \sin^{-1}(x^2 + y^2) \]