Sets and set notation

Sets of real numbers

- Natural (counting) numbers
- Whole numbers
- Integers
- Rational numbers
- Irrational numbers

are all subsets of the real numbers. For each category, be sure you can recognize what types of numbers belong in it. We won’t go through a formal construction of the reals. The lecture provides some basic information on who goes in what set and the notation; please look at the linked definitions for this one for more details.

- Natural (counting) numbers ($\mathbb{N}$)
- Whole numbers ($\mathbb{N} \cup \{0\}$)
- Integers ($\mathbb{Z}$)
- Rational numbers ($\mathbb{Q}$)

- Irrational numbers ($\mathbb{Q}'$)

- Real numbers ($\mathbb{R}$)
Intervals and interval notation

Several examples are presented here; be sure you can describe and sketch the sets below.

Examples:

• \( \{ x \mid x \in \mathbb{Z}, -2 < x \leq 3 \} \)

• \( \{ x \mid x \in \mathbb{R}, -2 < x \leq 3 \} \)

• \( \{ (x, y) \mid x \leq 2, y > -1 \} \)

• \( \{ (x, y) \mid x^2 + y^2 = 4 \} \)

• (open interval) \((a, b) = \{ x \mid x \in \mathbb{R}, a < x < b \} \)

• (closed interval) \([a, b] = \{ x \mid x \in \mathbb{R}, a \leq x \leq b \} \)
• \((-\infty, a) = \{ x \mid x \in \mathbb{R}, x < a \}\)

• \((b, \infty) = \{ x \mid x \in \mathbb{R}, x > b \}\)

• \([3, 7) = \{ x \mid x \in \mathbb{R}, 3 \leq x < 7 \}\)

• \((-\infty, 1) \cup [2, \infty) = \{ x \mid x \in \mathbb{R}, x < 1 \text{ or } x \geq 2 \}\)

• \([-1, 2) \cup (2, 4] = \{ x \mid x \in \mathbb{R}, -1 \leq x < 2 \text{ or } 2 < x \leq 4 \}\)
Mini-quiz:

(Q1) Which set is equal to the set shown below?

\{-1, 0, 1, 2, 3\}

(a) \{x \mid x \in \mathbb{Z}, -1 < x < 4\}
(b) \{x \mid x \in \mathbb{R}, -1 < x < 4\}
(c) \{x \mid x \in \mathbb{Z}, -1 \leq x < 4\}
(d) (-1, 4]

(Q2) Which set is equal to the set shown below?

(a) \{x \mid x \in \mathbb{Z}, -1 < x < 4\}
(b) \{x \mid x \in \mathbb{R}, -1 < x < 4\}
(c) \{x \mid x \in \mathbb{Z}, -1 \leq x < 4\}
(d) (-1, 4]

(Q3) Which set is equal to the set shown below?

(a) \{x \mid x \in \mathbb{Z}, -1 < x < 4\}
(b) \{x \mid x \in \mathbb{R}, -1 < x < 4\}
(c) \{x \mid x \in \mathbb{Z}, -1 \leq x < 4\}
(d) (-1, 4]

(Q4) Which set is equal to the set shown below?

\{0, 1, 2, 3\}

(a) \{x \mid x \in \mathbb{Z}, -1 < x < 4\}
(b) \{x \mid x \in \mathbb{R}, -1 < x < 4\}
(c) \{x \mid x \in \mathbb{Z}, -1 \leq x < 4\}
(d) (-1, 4]

(Q5) Which set is equal to the set shown below?

(a) \{x \mid x \in \mathbb{Z}, -1 < x < 4\}
(b) \{x \mid x \in \mathbb{R}, -1 < x < 4\}
(c) \{x \mid x \in \mathbb{Z}, -1 \leq x < 4\}
(d) (-1, 4]