Linear transformations

Compositions of transformations

Recall the composition of functions: suppose \( f(x) = 2x + 1 \), \( g(x) = x^2 \). What is the composition \( f \circ g = f(g(x)) \)?
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Compositions of linear transformations work the same way; we first apply one function, then the other. However, not all transformations can be composed with other; we have to make sure that the domains and codomains all line up.

Consider the transformations
\[
T_1 : \mathbb{R}^3 \rightarrow \mathbb{R}^5 \\
T_2 : \mathbb{R}^5 \rightarrow \mathbb{R}^2
\]
Does the composition \( T = T_1 \circ T_2 \) make sense? If so, what are the domain and codomain of \( T \)?

Does the composition \( T = T_2 \circ T_1 \) make sense? If so, what are the domain and codomain of \( T \)?
Example: Let

\[ T_1(<x, y>) = <x - y, x + y> \]
\[ T_2(<x, y>) = <2x, y, y - 3x> \]

- What are the domains and codomains of \( T_1 \) and \( T_2 \)?

- What are the domain and codomain of \( T = T_2 \circ T_1 \)?

- Find the image of the vector \(<-3, 4>\) under the transformation \( T \).

- Find an expression for \( T = T_2 \circ T_1 \).
If the transformations are represented by matrices, it’s easy to compose them. If $T_1(u) = A_1u$, and $T_2(v) = A_2v$, the composition $T = T_2 \circ T_1(u)$ is given by $A_2(A_1u) = A_2A_1u$.

First, put some dimensions on these things; what sizes do the matrices have to be for the above to make sense?

**Example:** Suppose

$$T_1(u) = \begin{bmatrix} 1 & 2 & -4 \\ 1 & 0 & 1 \end{bmatrix} u \quad T_2(v) = \begin{bmatrix} -1 & 3 \end{bmatrix} v$$

What is the transformation $T = T_2 \circ T_1$? Give domains and codomains for everything. Then, find the image of $u = <5, 1, -7>$ under the transformation $T$. 