Suggested problems

Norms on other spaces

P1: Consider the space of continuous functions on \([a, b]\), with inner product \(f(u(x), v(x)) = \int_{a}^{b} u(x)v(x) \, dx\). Find the expression that will give \(||g(x)||\) for \(g(x) = x^3\), where \(|| \cdot ||\) is the norm defined by the inner product.

P2: Let \(u\) and \(v\) be vectors in \(V\), where \(V\) is some vector space with an inner product \(f(u,v)\) and norm \(||v|| = \sqrt{f(v,v)}\). Prove that \(||u|| = ||v||\) if and only if \(u+v\) and \(u-v\) are orthogonal.

(The point to this one is you can prove properties without having any specific formula for norm or dot product in mind.) (Makes a nice final exam takehome question.)

P3: Define \(||f(x)|| := \max_{0 \leq x \leq 2} |f(x)|\) on the space of continuous functions.

(a) Prove it's a norm.
(b) Compute \(||f(x)||\) for \(f(x) = e^x\).
(c) Compute \(||f(x)||\) for \(f(x) = 5x - 7\)

P4: We don't really have any norms defined on spaces of matrices yet. We do have one function that takes a matrix and returns a scalar, though - the determinant. Consider \(|\det(A)|\) for \(A \in M_{22}\). [Notation: those are absolute value bars, which is why I'm using \(\det(A)\) and not \(|A|\) for determinant.] Explain why this function does not work as a norm on \(M_{22}\). Which properties does it pass (prove it), and which does it fail (give counterexample).

Takehome exam question. Guaranteed.