<table>
<thead>
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<th><strong>THE “STICK IT IN A MATRIX AND RREF IT” LIST</strong></th>
<th><strong>Set up</strong></th>
<th><strong>rref the augmented matrix</strong></th>
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</table>
| **Is a vector a linear combination of a given set?** | \( c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \ldots + c_m \mathbf{v}_m = \mathbf{v} \) | • unique or infinite – YES  
• no solution - NO |
| **Express a vector as a linear combination of a given set.** | Same as above. | Just continue to write the solution. |
| **Is a set of vectors linearly independent or dependent?** | \( c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \ldots + c_m \mathbf{v}_m = \mathbf{0} \) | rref the augmented matrix  
• unique – YES  
• infinitely many - NO |
| **Does a set span \( \mathbb{R}^n \)?** | \( c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \ldots + c_m \mathbf{v}_m = I \) | rref the augmented matrix  
• solution for every right hand side (column of I) – YES  
• no solution - NO |
| **Is a set a basis for \( \mathbb{R}^n \)?** | Fast check setup  
\( c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \ldots + c_m \mathbf{v}_m \)  
(just the coefficient matrix)  
Detailed answer – check independence and span | rref the coefficient matrix  
• identity – YES  
• not identity – NO  
 May also be able to use relevant theorems (must have \( n \) vectors in a basis for \( \mathbb{R}^n \)) |
| **Does a set span a subspace of \( \mathbb{R}^n \)?** | Set up  
\( c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \ldots + c_m \mathbf{v}_m = \mathbf{V} \)  
where \( \mathbf{V} \) is a matrix of vectors that are known to generate the space. | rref the augmented matrix  
• solution for every right hand side (column of \( \mathbf{V} \)) – YES  
• no solution - NO |
| **Does a set form a basis for a subspace of \( \mathbb{R}^n \)?** | Check independence and span as above. | Make sure all vectors are actually in the space first, if easy to verify form. |
| **Does a set span / is it independent / is it a basis for a space other than \( \mathbb{R}^n \)?** (matrices, polynomials) | Write the equivalent \( \mathbb{R}^n \) vector problem and solve. (2 x 2 matrices \( \rightarrow \) \( \mathbb{R}^4 \) vectors, polys of degree 3 or less \( \rightarrow \) \( \mathbb{R}^3 \) vectors, etc.) |  |
| **Basis for the row space of \( A \).** | rref(\( A \)). Non-zero rows are basis vectors. rank(\( A \)) = dimension of row space = number of non-zero rows. | Only time you’ll read the rows of a matrix as vectors. |
| **Basis for the column space of \( A \).** | rref(\( A^\top \)). Row space of \( A^\top \) (transpose) = column space of \( A \). |  |