Suggested problems - solutions

Defining the dot product

P1: Compute \( \mathbf{u} \cdot \mathbf{v} \) for \( \mathbf{u} = \langle 2, -4 \rangle \) and \( \mathbf{v} = \langle 3, 5 \rangle \).

\[
\mathbf{u} \cdot \mathbf{v} = \langle 2, -4 \rangle \cdot \langle 3, 5 \rangle = 2(3) + (-4)(5) = 6 - 20 = -14
\]

The most common mistake with the dot product is to forget it’s a scalar and not a vector - be sure to add all the bits into a single value [e.g. it’s 6 PLUS -20, not 6 COMMA -20]. Additional problems of this type at COW:

3. Calculus Book III
   > 2. Vectors and Analytic Geometry
   > 2. Dot Product in the Plane
   > 1. Dot Product in the Plane

P2: Compute \( \mathbf{u} \cdot \mathbf{v} \) for \( \mathbf{u} = \langle 1, -4, 7 \rangle \) and \( \mathbf{v} = \langle 1, 0, -1 \rangle \).

\[
\mathbf{u} \cdot \mathbf{v} = \langle 1, -4, 7 \rangle \cdot \langle 1, 0, -1 \rangle = 1(1) + (-4)(0) + 7(-1) = 1 + 0 - 7 = -6
\]

Additional problems of this type at COW:

3. Calculus Book III
   > 2. Vectors and Analytic Geometry
   > 3. Dot Product in Three Dimensions
   > 1. Dot Product in Three Dimensions

P3: Prove the property of the dot product for \( \mathbf{u}, \mathbf{v} \) in \( \mathbb{R}^n \), \( c \) scalar:

\[
c \mathbf{u} \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (c \mathbf{v})
\]

Let \( \mathbf{u} = \langle u_1, u_2, ..., u_n \rangle \), \( \mathbf{v} = \langle v_1, v_2, ..., v_n \rangle \), \( c \) scalar.

\[
c \mathbf{u} \cdot \mathbf{v} = \langle cu_1, cu_2, ..., cu_n \rangle \cdot \langle v_1, v_2, ..., v_n \rangle = cu_1v_1 + cu_2v_2 + ... + cu_nv_n = c(u_1v_1 + u_2v_2 + ... u_nv_n) = c(\mathbf{u} \cdot \mathbf{v})
\]

Furthermore, since for each term \( cu_i v_i \), we can say that \( cu_i v_i = u_i(cv_i) = u_i(cv_i) \), and the above is also equal to

\[
u_1(cv_1) + u_2(cv_2) + ... + u_n(cv_n) = \mathbf{u} \cdot (c \mathbf{v})
\]